



**Physics Olympiad «Phystech»,
February 2023**

Problem Set 10-03



Ordinary fractions and radicals are allowed in your answers in all the problems of the set.

1. A projectile flies vertically and explodes at the highest point of the trajectory into many fragments flying in all possible directions with equal velocities in modulus. In $t_1 = 0.4$ s after the explosion all the fragments are in flight, one of the fragments moves horizontally, its momentum is $P_1 = 30$ kg·m/s. The mass of the projectile is $M = 10$ kg.

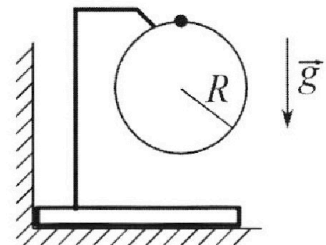
1) Find the module P_2 of the total momentum \vec{P}_2 of all the other fragments at this time point. Acceleration due to gravity is $g = 10$ m/s².

2) Find the angle α between the vectors \vec{P}_2 and \vec{g} at this time point. In the answer specify the value of the trigonometric function of the angle α : $\sin \alpha$ or $\tan \alpha$.

The maximum distance from the point of explosion to the ground point of impact equals $d = 80$ m.

3) Find the duration T of the flight of such fragments. Air resistance can be ignored.

2. The bar is installed close to the vertical wall (see Fig.). The ring of radius $R = 1$ m is fixed on the bar (the ring is in the vertical plane). A ball is put on the ring. The masses of the bar and the ball are the same. The ring and the holder are lightweight. There is no friction. From the upper point of the ring the ball slides with a negligible initial velocity.



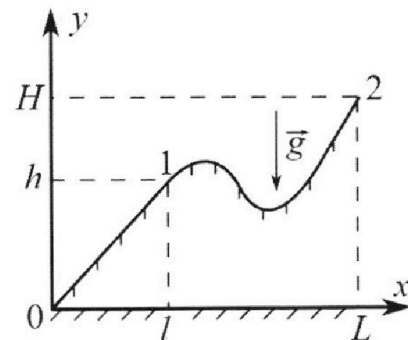
1) Find the acceleration \vec{a} of the ball at the moment when the force with which the bar acts on the vertical wall turns to zero. In the answer specify the module and direction of the vector \vec{a} .

2) Find the vertical displacement h of the ball to this point of time.

3) Find the maximum speed V of the bar movement.

Acceleration due to gravity is $g = 10$ m/s². In the process of movement the bar does not break away from the smooth horizontal surface.

3. A schoolboy pulls a sled on a hill moving in the straight line. Mass of the sled is $m = 5$ kg. The profile of the hill in the vertical plane is shown in the drawing for the task. In order to pull the sled slowly from point 0 to point 1, applying force along the flat surface of the hill, it is necessary to perform work $A_1 = 300$ J. At point 1 the schoolboy releases the sled. The vertical coordinate of the starting point is $h = 4.6$ m, the initial velocity of the sled is zero. The kinetic friction coefficient is the same on the entire surface of the hill. Acceleration due to gravity is $g = 10$ m/s².



1) Find the speed V of the sled at the base of the hill at point 0.

2) Find the work A_2 that should be done to move the sled slowly from point 1 to point 2? The vertical coordinate of the point 2 is $H = 10$ m, $L = 4l$. At each elementary displacement the vector of the force that the schoolboy applies to the sled and the vector of the sled displacement lie on the same straight line. All displacement occur in the same vertical plane.



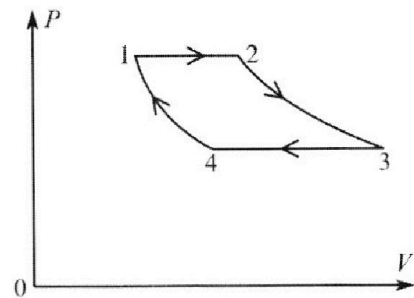
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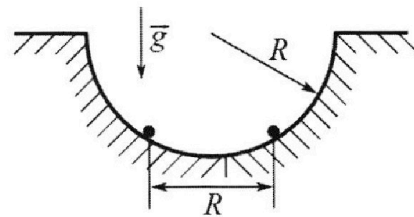


4. In the 1-2-3-4-1 cycle of the heat engine, there are two isobars and two isotherms (see Fig.). The working substance is a monatomic ideal gas. In the process of isobaric expansion up to double the volume, the gas performs the work A . The gas performs the same work A during isothermal expansion.



- 1) Find the amount Q of heat added to the gas in processes 1-2-3.
- 2) Find the amount Q_{34} of heat removed from the gas in the process of isobaric compression ($Q_{34} > 0$).
- 3) Find the efficiency e of the cycle.

5. A hemispherical hole of radius R is made in a smooth horizontal surface. Two charged balls are held in the hole at the same horizontal level (see Fig.). The mass of each ball is m , the distance between the balls is R . The balls are simultaneously released and they come off the hole at the edges. There is no friction. Counted from the edge of the hole the maximum height to which each ball rises in flight is equal to R .



- 1) Find the speed V of each ball in the beginning of the flight.
- 2) Find the charge Q of each ball.
- 3) Find the maximum speed U with which the distance between the balls grows after beginning the flight.

Acceleration due to gravity is g . The collisions of the balls with the horizontal surface are absolutely elastic. The proportionality coefficient in Coulomb's law is k .



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MIPT

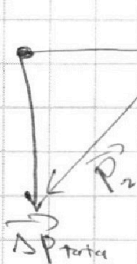
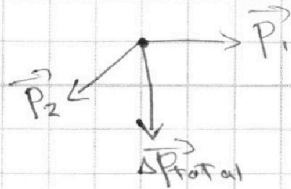
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ΔP_{total} is the change in momentum for all the projectile

$$\Delta P_{total} = Mg t_1 = 10 \times 10 \times 0.4 = 40 \text{ kg.m/s}$$

(downward)

1)



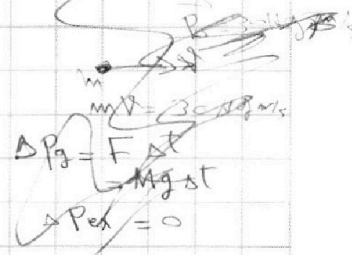
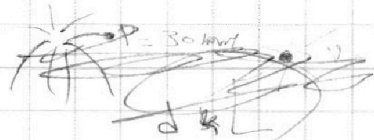
$$\Delta P_{total} = P_1 + P_2$$

$$\Rightarrow P_2 = \Delta P_{total} - P_1$$

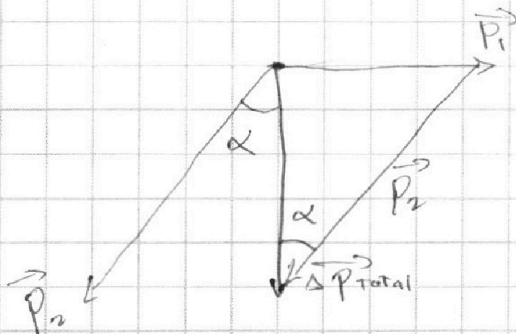
$$P_2 = \sqrt{P_{total}^2 + P_1^2}$$

$$= \sqrt{40^2 + 30^2} = \sqrt{1600 + 900} = 50 \text{ kg.m/s}$$

$$P_2 = 50 \text{ kg.m/s} \text{ Answer}$$



2)



$$\tan \alpha = \frac{P_1}{\Delta P_{total}} = \frac{30}{40} = \frac{3}{4}$$

$$\tan \alpha = \frac{3}{4} \text{ Answer}$$

3) ~~v_0~~ v_0 = The initial velocity of the projectile

~~$$v = v_0 - gt, \text{ at highest point } 0 = v_0 - \frac{gT}{2}$$~~

~~$$y = -\frac{1}{2}gt^2 + v_0t, \text{ at highest point } 80 = -\frac{1}{2}g\frac{T^2}{4} + v_0\frac{T}{2}$$~~

~~$$v_0 = \frac{gT}{2} \Rightarrow 80 = -\frac{1}{2}g\frac{T^2}{4} + \frac{gT^2}{4} = \frac{gT^2}{8} \Rightarrow T = \sqrt{\frac{8 \times 80}{g}}$$~~

~~$$T = \sqrt{8 \times 8} = 8 \text{ s} \text{ Answer}$$~~



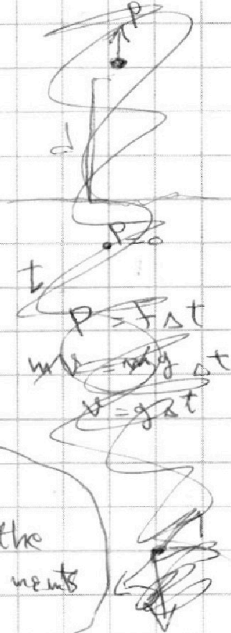
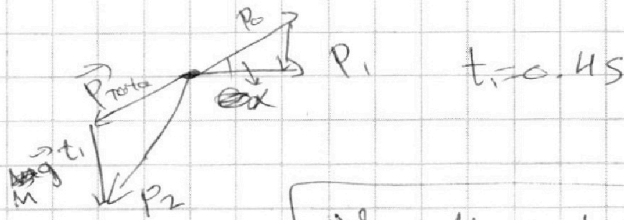
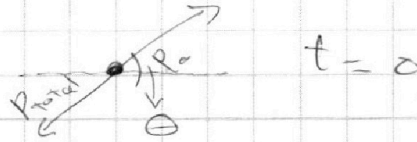
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3) The total momentum at the instant of explosion is zero



~~$\tan \alpha = \tan \theta$~~
 V_0 : the initial velocity for the fragments

$$\tan \theta = \frac{p_1 + \Delta P_{\text{Total}}}{\Delta P_{\text{Total}}} = \frac{30 + 40}{40} = \frac{7}{4}$$

$$V_0 \sin \theta = g t_1$$

$$V_0 = \frac{g t_1}{\sin \theta} = \frac{10 \times 0.4 \sqrt{65}}{7}$$

~~$\frac{1}{2} g T_1^2$~~ $V_i = \text{initial projectile velocity}$

(at top) $0 = V_{i1} - g T_1$

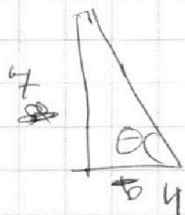
$$80 = -\frac{1}{2} g T_1^2 + V_{i1} T_1$$

$$80 = -\frac{1}{2} g T_1^2 + g T_1^2 \Rightarrow T_1 = 4 \text{ s}$$

$$-\frac{1}{2} g T_2^2 + V_{i2} T_2 = -80 \Rightarrow$$

$$T_2 = \frac{-V_{i2} + \sqrt{V_{i2}^2 + 4 \times 80 \times 5}}{-10} \approx 4.6$$

$$\Rightarrow \sin \theta = \frac{7}{\sqrt{7^2 + 4^2}} = \frac{7}{\sqrt{49 + 16}} = \frac{7}{\sqrt{65}}$$





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MIPT

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3) Δp_x : the total momentum
on ~~ax~~ [the horizontal]
thier is no external force acting
on the horizontal plane
 $\Rightarrow \Delta p_x = 0 \Rightarrow p_x = 0$ ~~the center mass~~
~~is not moving~~
 ~~$V_{bar} = V \cos \theta$~~

$$V = V \cos \theta = \sqrt{\frac{20}{3}} \cdot \frac{2}{3} = \sqrt{\frac{10}{3}} \text{ m/s}$$

$V = \sqrt{\frac{10}{3}} \text{ m/s}$



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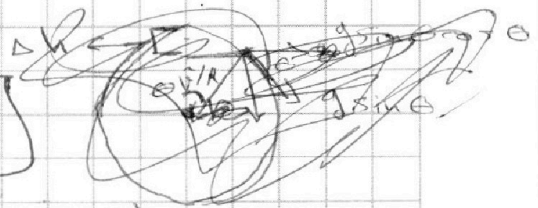
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$$1) \quad g \sin \theta \cos \theta = \frac{v^2}{R} \sin \theta$$

$$\cos \theta = \frac{v^2}{gR}$$

v : ball
Velocity



$$\Delta h = R - R \cos \theta = R(1 - \cos \theta)$$

$$mg \Delta h = \frac{1}{2} mv^2 \Rightarrow v = \sqrt{2g \Delta h}$$

$$v = \sqrt{2gR(1 - \cos \theta)}$$

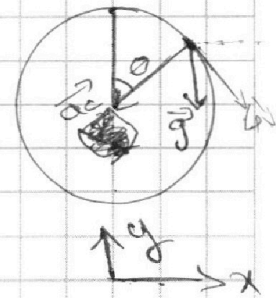
~~$gR \cos \theta = \frac{2gR(1 - \cos \theta)}{R} \Rightarrow \cos \theta = 2(1 - \cos \theta) \Rightarrow \cos \theta = 2 - 2 \cos \theta \Rightarrow 3 \cos \theta = 2 \Rightarrow \cos \theta = \frac{2}{3}$~~

$$gR \cos \theta = 2gR(1 - \cos \theta)$$

$$\cos \theta = 2 - 2 \cos \theta$$

$$\cos \theta = \frac{2}{3}$$

$$v = \sqrt{2 \times 10 \times 1 \times \left(1 - \frac{2}{3}\right)} = \sqrt{\frac{20}{3}}$$



$$|a| = \sqrt{\left(\frac{v^2}{R}\right)^2 + (g \sin \theta)^2}$$

~~$= \sqrt{\frac{400}{9} + \frac{100}{9}} = \sqrt{\frac{500}{9}}$~~

$$= \sqrt{\frac{400}{9} + 100 \times \left(1 - \frac{4}{9}\right)} = \sqrt{\frac{400}{9} + \frac{500}{9}}$$

$|a| = 10 \text{ m/s}^2$ Answer

directed downward



$$2) \quad h = R(1 - \cos \theta) = \frac{R}{3} = \frac{1}{3} \text{ m}$$

$h = \frac{1}{3} \text{ m}$ Answer



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1) E_p : the potential energy

$$E_p = mgh = 5 \times 10 \times 4.6 = 230 \text{ J}$$

W_f : the work done by the friction force

$$A_1 = W_f + E_p \Rightarrow W_f = A_1 - E_p = 300 - 230 = 70 \text{ J}$$

the same friction force will be applied while travelling down ward

E_k : the kinetic energy of the ball at the end of the hill

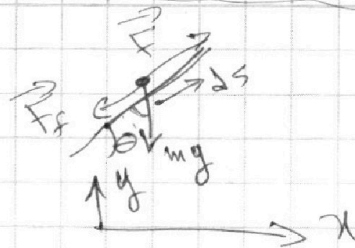
$$E_k = E_p - W_f = 230 - 70 = 160 \text{ J}$$

$$\frac{1}{2}mv^2 = 160 \text{ J} \Rightarrow v = \sqrt{\frac{2 \times 160}{m}} = \sqrt{\frac{320}{5}} = \sqrt{64} = 8 \Rightarrow \boxed{v = 8 \text{ m/s}} \text{ Answer}$$

~~this~~ F_f : friction force

ds : elementary displacement

$$F_f = mg \cos \theta, \text{ for elementary work } dW, dW = F_f ds = mg \cos \theta ds = mg dx$$



then the friction work depend only on the x

Horizontal component.

~~$A_1 = 300 \text{ J}$~~
 ~~$A_2 = 480 \text{ J}$~~

W_{f2} : friction work
 $W_{f2} = 3W_{f1} = 3 \times 70 = 210$

$$A_2 = W_{f2} + E_{p2} = 210 + mg(H-h) = 210 + 5 \times 10 \times 5.4 = 210 + 270 = 480 \text{ J}$$

$$\boxed{A_2 = 480 \text{ J}} \text{ Answer}$$



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~~$\Delta E_{int} = Q - W$~~

1) $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, $P_1 V_1 = nRT_1$
 $P_1 (2V_1) = nRT_2$
 $T_2 = 2T_1$
 $Q_{12} = n C_P \Delta T$
 $= n C_P (T_2 - T_1)$

$A = P \Delta V = P_1 V_1 = nRT_1 \Rightarrow T_1 = \frac{A}{nR}$

$Q_{12} = \frac{5}{2} nR \frac{A}{nR} = \frac{5}{2} A$

ΔE_{int} : the total internal energy

$T_2 = T_3 \Rightarrow \Delta E_{int} = 0 \Rightarrow Q_{23} = A$

$Q_{1-2-3} = Q_{12} + Q_{23} = \frac{5}{2} A + A = \frac{7}{2} A$

$Q_{123} = \frac{7}{2} A$ Answer

2) $T_4 = T_1$, $T_3 = T_2$

$Q_{34} = n C_P \Delta T = n C_P T_1 = \frac{5}{2} nR \frac{A}{nR} = \frac{5}{2} A$

$Q_{34} = \frac{5}{2} A$

3) W_{34} : work done by the gas during $3 \rightarrow 4$

$W_{34} = -Q_{34} - \Delta E_{int} = -\frac{5}{2} A - n C_V \Delta T$

$= -\frac{5}{2} A + \frac{3}{2} nR T_1 = -\frac{5}{2} A + \frac{3}{2} A = -A$ $W_{34} = -A$

$Q_{41} = W_{41} = nRT_1 \ln \left| \frac{V_1}{V_4} \right|$



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$$3) P_2 V_2 = P_3 V_3 \Rightarrow 2 P_1 V_1 = P_3 V_3$$

$$nRT_2 \ln \left| \frac{V_3}{V_2} \right| = A$$

$$2nRT_1 \ln \left| \frac{V_3}{V_2} \right| = A$$

$$2 \frac{nR A}{nR} \ln \left| \frac{V_3}{2V_1} \right| = A \Rightarrow \ln \left| \frac{V_3}{2V_1} \right| = \frac{1}{2}$$

$$\boxed{V_3 = 2V_1 e^{1/2}}$$

$$W_{34} = -A = P_3 (V_4 - V_3)$$

$$V_4 = -\frac{A}{P_3} + V_3 = -\frac{A}{2nRT_1} + V_3 = -\frac{V_3}{2} + V_3$$

$$\cancel{V_3 = 2V_4} \quad V_4 = \frac{1}{2} V_3 = V_1 e^{1/2}$$

$$Q_{41} = W_{41} = nRT_1 \ln \left| \frac{V_1}{V_1 e^{1/2}} \right| = -A \ln |e^{1/2}|$$

$$\boxed{Q_{41} = -\frac{A}{2}}$$

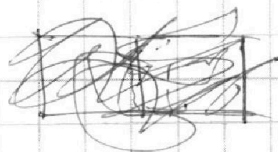
$$\boxed{W_{41} = -\frac{A}{2}}$$

$$\Sigma W = 2A - A - \frac{A}{2} = \frac{A}{2}$$

$$\Sigma Q = \frac{7}{2}A - \frac{5}{2}A - \frac{A}{2} - \frac{A}{2}$$

$$\boxed{e = 1}$$

$$\leftarrow e = \frac{W_{eng}}{Q_{in}}$$



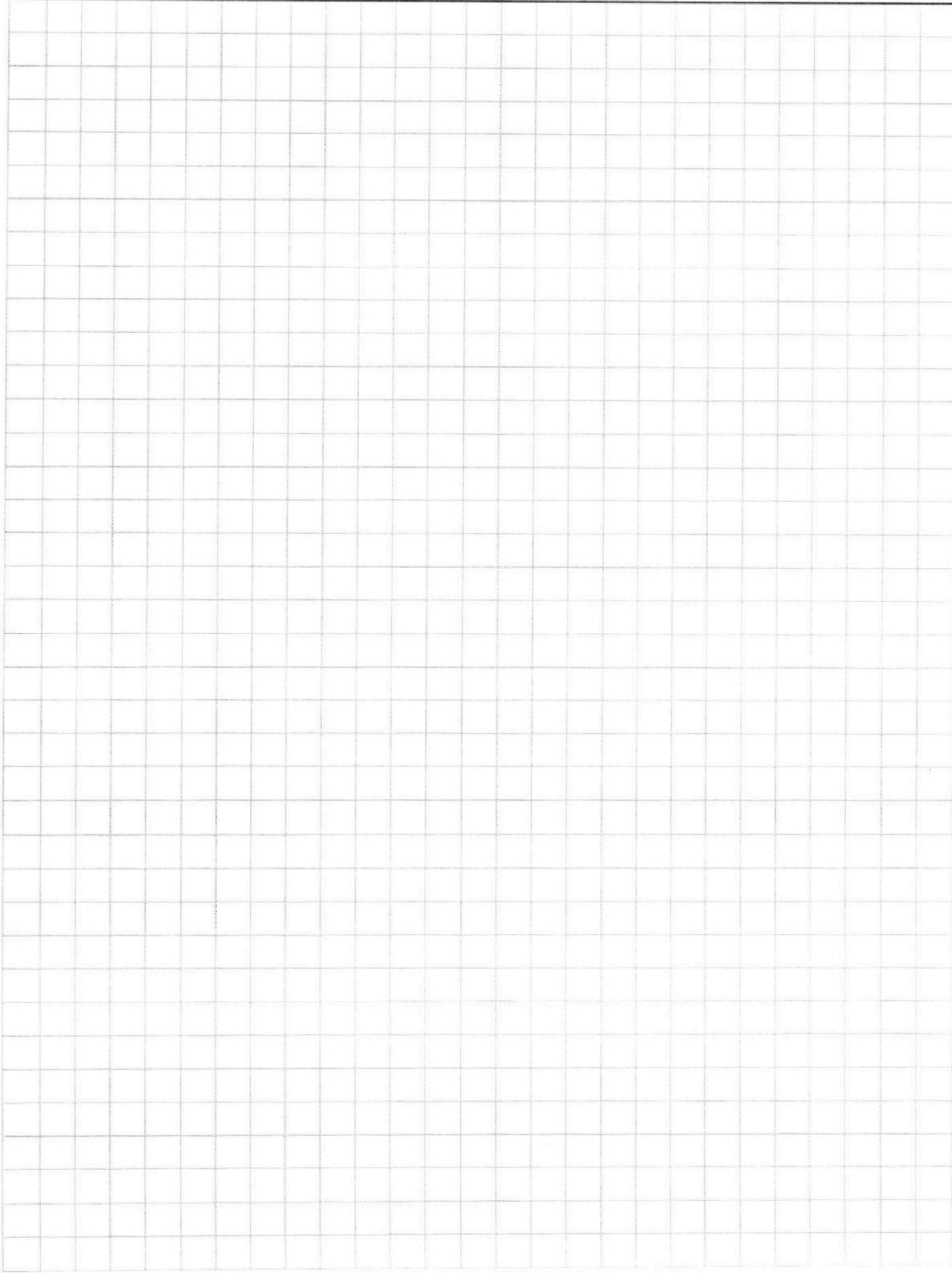


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$$\frac{-4 \pm \sqrt{65}}{7} + \sqrt{\frac{16}{49} + 1600}$$

-10

$$\approx \frac{-32}{7} + 40 = \frac{-4 \cdot 3 + 40}{-10}$$



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